PATENT

METHOD FOR MODELING VOLATILITY

FIELD OF THE INVENTION

The present invention relates to financial risk management. Particularly, the invention relates to modeling the volatility of a swap option.

BACKGROUND

Swap options are associated with a volatility. There are various methods of estimating the volatility of a swap instrument. The current standard method for pricing swaptions is the Black-Scholes formula. The Black model takes a single volatility as its input. The Black formula assumes that interest rates are lognormally distributed. Accordingly, as the strike rate (exercise rate) varies from the forward rate (the projection of future rates) in the Black formula, the resultant implied volatility remains constant. However, as market observations show, the implied volatility of an option changes as the strike rate moves away from the forward rate underlying the option. Therefore, there is a need for a method for properly valuating swap options while recognizing that the options sometimes have different strike and forward rates.

SUMMARY OF THE INVENTION

Accordingly, the present invention provides a method for valuating swap options that recognizes that implied Black volatilities vary when strike rates differ from forward rates. The method of the invention allows for a correlation between the forward rate and its volatility by employing the "q-model" (discussed below). The method employs two intuitive parameters, volatility of volatility and q, determined by a trader, to produce a model for the non-flat volatility curve.

The method includes providing the average volatility of the asset by employing market data and providing the volatility of volatility of the asset by employing historical data. The method further includes providing the type of distribution for the forward rate based on historical data. A volatility distribution graph is provided, based on the selected distribution type, the graph having volatility as the x-axis and probability as the y-axis. The method then divides the volatility distribution graph into a plurality of vertical slices, each of said slices corresponding to a volatility such that the integration of the graph over the volatility range corresponding to each slice provides a probability for the corresponding volatility. The method determines a stochastic volatility premium for each volatility by employing a volatility premium process. The method then weighs each premium from said determining of premium step by the probability associated with the corresponding volatility, as determined from the volatility distribution graph. Finally, the method sums all premiums associated with the volatilities to provide a premium for the option.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 is a flow diagram illustrating a process for determining a swap option volatility in accordance with the invention;

Figure 2 illustrates a stochastic distribution of interest rate volatility chart; and

Figure 3 illustrates the Black volatility model and the model of the invention relative to actual market volatility.

DETAILED DESCRIPTION

The method for pricing options in accordance with the invention will be discussed with reference to a stochastic distribution curve and corresponding flow diagram

outlining a method for determining the volatility, and in turn the price, of swap options. Next, a model in accordance with the invention is compared to the Black model by illustrating the shape of typical volatility curves arrived at by each method.

One method for modeling the volatility of a swap option is by employing the Hull and White stochastic volatility model. The Hull and White model determines the price of option by creating a lognormal distribution of asset price variance. The Hull method shows that, in the absence of correlation between rates and variance, the price of a European option (fixed date of exercise) is the Black price integrated over the probability of distribution of the variance of the rate over the life of the option. The method of the present invention extends the Hull method by creating a lognormal distribution of the volatility instead of the variance and incorporating non-zero correlation between volatilities and rates.

Figure 1 is a flow diagram illustrating the process for pricing options in accordance with the invention. Average volatility as well as volatility of volatility are determined either by calibration to the values as seen in the market or by reference to historical data relating to the swap asset (Step 22). The historical data is preferably processed by a trader to determine the average volatility and volatility of volatility. One method for determining the average volatility is by using the existing implied volatility that is quoted in the market. One method for determining the volatility of volatility is by calculating a standard deviation from a history of implied volatility values. A trader preferably selects a distribution type for the volatility by employing historical data relating to the swap asset or by calibrating to the values implied in the market (Step 20).

In one embodiment, the distribution type is selected by estimating the relationship from historical volatility and yield data.

Figure 2 represents a volatility probability curve 34 constructed in step 24 of the method illustrated in Figure 1. A volatility distribution curve 34 is constructed using the average volatility and volatility of volatility determinations in combination with the selected distribution type. The volatility of rates preferably corresponds to the x-axis of the graph. The probability of volatility preferably corresponds to the y-axis of the graph. The average volatility is the weighted center of the curve 34. The graph is then divided into a plurality of vertical slices, as is illustrated in Figure 2 (Step 26). The area under the curve corresponding to each slice represents the total probability of volatility for the particular volatility associated with the slice.

Referring back to Figure 1, an option premium is calculated for each slice and is weighed by the corresponding probability (Step 28). In one embodiment, an option premium is calculated for each volatility by employing the "q-model." The "q-model" describes a formula for determining a stochastic volatility premium for a swap option. Q-model value of a call option on rate r with forward value $\bar{r}$, strike k, expiration time t, and annualized volatility σ is given by the following formula:

$$BSQ(\bar{r},c,\sigma,t) = \bar{r}\tfrac{1}{q}\cdot\Phi(d_1) + \bar{r}(1-\tfrac{1}{q}-\tilde{k})\cdot\Phi(d_2)$$

where Φ is the normal cumulative inverse function and

$$\tilde{k} = k/\bar{r}$$
$$\tilde{x} = -\tfrac{1}{q}\ln[(\tilde{k}-1)q+1]/(\sigma\sqrt{t})$$
$$d_1 = \tilde{x} + \tfrac{1}{2}q\sigma\sqrt{t}$$
$$d_2 = \tilde{x} - \tfrac{1}{2}q\sigma\sqrt{t}$$

The q-model calculates an option premium by linearly interpolating between a lognormal and normal distribution of rates. The probability associated with each volatility as derived from the graph of Figure 2 is used to weigh each q-model premium. The premiums are then summed up to produce the stochastic volatility premium (Step 30). After the stochastic volatility premium is determined, the volatility premium is optionally inverted back to a standard market volatility by using the known Black method.

For the strike rate that is the same as forward rate, or "at the money," the method of the invention will produce a volatility that is the same as the one produced by the Black model. However, for strike rates that are not equal to the forward rate, the method of the present invention produces a convex curve that is different from the flat curve produced by the Black method.

Figure 3 illustrates volatility curves for the Black method, the method of the invention, and actual market volatility. As may be appreciated the volatility curve produced by the method of the invention 38 is much closer to the market volatility curve 36 then the Black curve 40 when the strike rate moves away from the forward rate. The convex shape of the curve produced by the method of the invention 38 is the result of integrating a lognormal distribution of volatility, where the distribution of the underlying forward rate is a linear interpolation between a lognormal and normal distribution. The magnitude of the dispersion of the distribution of volatility determines the convexity of the curve 38. The distribution of the forward rate (i.e., whether it is lognormal, normal, or somewhere in-between) determines the slope of the curve 38.

Although the present invention was discussed in terms of certain preferred embodiments, the description is not limited to such embodiments. Rather, the invention includes other embodiments including those apparent to a person of ordinary skill in the art. Thus, the scope of the invention should not be limited by the preceding description but should be ascertained by reference to the claims that follow.